BEST POST-TRANSFORMS SELECTION IN A RATE-DISTORTION SENSE

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ABSTRACT

This paper deals with the optimization of a new technique of image compression. After the wavelet transform of an image, blocks of coefficients are further linearly decomposed using a basis selected in a dictionary. This dictionary is known by both the encoder and the decoder. This approach is a generalization of the bandelet transform. This paper investigates the problem of the best basis selection. On each block of wavelet coefficients, this selection is made by minimization of a Lagrangian rate-distortion criterion. Theoretical expressions of the optimal Lagrangian multiplier can be computed based on asymptotic hypotheses. A nearly exhaustive search of the optimal Lagrangian multiplier is done for the compression of high resolution satellite images. This numerical study validates the asymptotic theoretical expressions but as well provides a refined expression of the Lagrangian multiplier. At last, the compression results obtained using those different expressions are compared to the optimal compression results obtained with the exhaustive search.

Index Terms**—** Image coding, wavelet transforms, discrete transforms, optimization methods, satellite applications

1. INTRODUCTION

Wavelet transform has become the common way to achieve very efficient still image compression. This transform is used in JPEG2000 standard as well as in the new CCSDS recommendation for image data compression [1] which specially targets on-board spacecraft compression. Thanks to the pyramidal decomposition which gives an efficient intrinsic organization of the information, powerful embedded coders have been designed such as EBCOT [2] (Embedded Block Coder with Optimal Truncation points) in JPEG2000 or the BPE [1] (Bit-Plane Encoder) in the CCSDS recommendation. Those coders exploit the information redundancy that still exists between adjacent wavelet coefficients in space or in scale.

Recently, a new approach has been proposed by Peyré in [3]. Instead of exploiting the information redundancy at the coding time, a post-transform takes advantage of the residual directional correlations between wavelet coefficients in a small neighborhood. This post-transform is called bandelet transform by groupings. Post-transforms are linear transforms applied on small blocks of wavelet coefficients. They are selected for each block among a dictionary of bases known by both the encoder and the decoder. A post-transform is applied on a given block only if it provides a representation easier to compress i.e. a better representation in the rate-distortion sense.

This paper investigates the best basis selection for each block of the wavelet transform. The bases in the dictionary have been built by PCA (Principle Component Analysis). This dictionary has been proposed in [4] which addresses the problem of bases construction for the compression. Compression results comparison between the post-transforms and JPEG2000 can also be found in [4]. The empirical methods used to find the best bases are adapted from the method described in [5] for the problem of the best bit budget allocation given a set of quantizer and from the method described in [6] for the problem of the best wavelet-packet decomposition. In section 2, the post-transform compression scheme is reviewed and the problem of the best basis selection is introduced. In section 3, two theoretical expressions of the Lagrangian multiplier which optimize the basis selection are given. One is extracted from [7] under the low bit rate assumption. The other one is derived from the high resolution hypothesis. In section 4, the optimal Lagrangian multipliers for the compression of satellite images are obtained by a nearly exhaustive search. The theoretical expressions are compared to these results and a new empirical expression is given. Finally, in section 5, compression results obtained using these different expressions of the Lagrangian multipliers are compared to the compression results obtained using the optimal Lagrangian multipliers found by the exhaustive search.

2. POST-TRANSFORM APPROACH

The compression scheme used in this paper has first been proposed by Peyré in [3] and is fully explained in [4]. After the wavelet transform of the image, post-transforms are applied on each block of 4×4 wavelet coefficients. This block size is the best for a simple and effective compression. Furthermore, studies have shown that correlations between nearby wavelet coefficients are very low at a distance greater than 4 pixels [8]. Here the dictionary of 36 bases contains 12 orthonormal bases for each orientation (HL, LH or HH) of the wavelet decomposition. They are obtained from PCA on different sets of blocks of wavelet coefficients as described in [4]. These bases perform a very efficient decorrelation of wavelet coefficients.

The figure 1 presents the post-transform process. For each block i of wavelet coefficients denoted by f_i, all the post-transforms associated to the bases $b \in [1, N_{\mathcal{B}}]$ in the dictionary are tested. On each block, the best basis b^* which gives the post-transform representation f_i^{b*} which minimizes the rate r_i^b and distortion d_i^b trade-off $d_i^b + \lambda r_i^b$ is selected for the compression. This selection method has been introduced in [7] for the first version of the bandelet transform. The problem is to find the Lagrangian multiplier λ which optimizes the compression efficiency.

The measure of the distortion $d_i^b = \|f_i^b - f_{i\Delta}^b\|$ is the mean square error between a post-transformed representation of the block before and after the quantization. As explained in [4], the bit rate r_i^b on each block is estimated based on the distribution of the wavelet

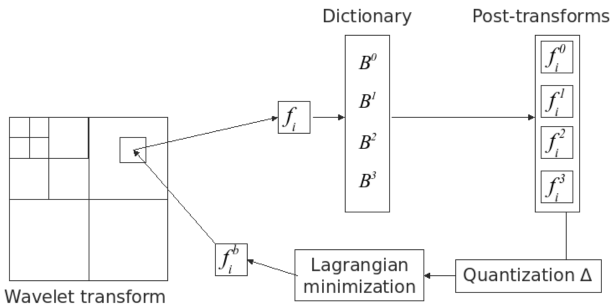

Fig. 1. The post-transform scheme.

coefficient in each subband. Moreover, the additional cost required to signal the selected basis b for each block is included in this bit rate estimation. In this paper, once the best post-transform representations of each block have been selected, an adaptive arithmetic coder is used to compress the coefficients as well as the identifiers of the selected bases. Given a quantization step Δ, the goal is to minimize the overall rate-distortion cost:

$$D(\Delta) + \lambda R(\Delta) \tag{1}$$

where $D(\Delta) = \sum_i d_i^{b*}$ and $R(\Delta)$ is the total bit rate.

3. THEORETICAL LAGRANGIAN MULTIPLIERS

In [7], Le Pennec addresses the problem of the optimal Lagrangian multiplier for the best bandelet basis selection. The goal is to minimize the Lagrangian cost (1) where the distortion D and the bit rate R are related to the quantization step Δ. When the Lagrangian cost (1) is minimized, its derivative vanishes:

$$\frac{\partial D}{\partial \Delta} + \lambda \frac{\partial R}{\partial \Delta} = 0 \tag{2}$$

3.1. Expression under low resolution hypothesis

An expression of the variation of the distortion D with the variation of the quantization step Δ is given in [7]. It depends on the variation of the number M of non zero wavelet coefficients:

$$\frac{\partial D}{\partial \Delta} \approx -\frac{3\Delta^2}{4} \frac{\partial M}{\partial \Delta} \tag{3}$$

This expression holds under the low bit rate assumption and for a uniform scalar quantization outside the zero bin which is twice larger than the others. This is a common quantization for wavelet compression [1, 2]. An approximation of the bit rate R under the low bit rate assumption is extracted from [9]. It also depends on M:

$$R \approx \gamma_0 M \quad \text{with } \gamma_0 = 6.5 \tag{4}$$

Finally, the following Lagrangian multiplier expression is obtained by combining the equations (2-4).

$$\lambda \approx \frac{3}{4\gamma_0} \Delta^2 \tag{5}$$

3.2. Expression under high resolution hypothesis

Under the high resolution hypothesis, a similar expression can be obtained. Indeed, for a high resolution uniform quantizer, the mean square error D is approximated by

$$D \approx \frac{\Delta^2}{12} \tag{6}$$

and the average bit rate R can be expressed with

$$R \approx h(X) - \log_2 \Delta \tag{7}$$

where $h(X)$ is the differential entropy of the wavelet coefficients of the image and thus, is a constant for each image. Differentiating equations (6) and (7) with respect to Δ and combining these expressions in equation (2) leads to another approximation of the Lagrangian multiplier:

$$\lambda \approx \frac{\ln(2)}{6} \Delta^2 \tag{8}$$

Although the hypotheses are different, the two approximations (5) and (8) of the Lagrangian multiplier are similar. They both express λ as a linear function of the square of the quantizer step Δ^2. Furthermore, in both cases, the multiplier factor is approximately 0.115. In order to verify the validity of these theoretical expressions, an exhaustive search of the optimal Lagrangian multipliers for the compression of several satellite images is executed in the next section.

4. EMPIRICAL LAGRANGIAN MULTIPLIER

The nearly exhaustive search of the optimal Lagrangian multiplier is made using processes similar to the ones described in [5, 6].

4.1. Best basis selection

In this section, the quantizer step Δ is fixed. Each block i of wavelet coefficients f_i is transformed using all possible bases $b \in [1, N_{\mathcal{B}}]$ in the dictionary. For each post-transformed representation f_i^b of the block f_i the distortion d_i^b and an estimation r_i^b of the bit rate needed to encode the quantized representation of f_i^b are computed. Given any value of the Lagrangian multiplier, the following algorithm is used to select the optimal representation of each block. Thus, it also computes the optimal rate $R^*(\lambda)$ and distortion $D^*(\lambda)$ for the fixed λ and Δ.

Algorithm 1: Optimal rate-distortion point

Input: The Lagrangian multiplier λ and the rate-distortion points $\left\{\left(r_i^b, d_i^b\right)\right\}_{b,i}$ of each block i transformed in each basis b

Output: The optimal rate $R^*(\lambda)$ and distortion $D^*(\lambda)$

foreach *block* i **do**

```
// Select the representation which
   minimizes the Lagrangian cost
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$b^* = \arg\min_{b \in [1, N_{\mathcal{B}}]} \left(d_i^b + \lambda\, r_i^b\right)$

end

$R^*(\lambda) = \sum_i r_i^{b*}(\lambda)$ and $D^*(\lambda) = \sum_i d_i^{b*}(\lambda)$

Figure 2 is a graphical interpretation of the best basis selection process on one block f_i. The best representation for the given slope $-\lambda$ is found by minimizing the cost $d_i^b + \lambda\, r_i^b$.

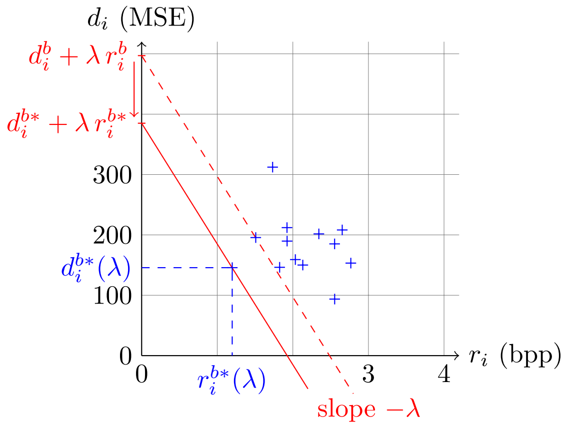

Fig. 2. Graphical interpretation of the best basis selection process on one block. Each cross corresponds to the rate-distortion point (r_i^b, d_i^b) of the quantized representation of f_i^b. The best basis for the given slope $-\lambda$ is found by minimizing the quantity $d_i^b + \lambda\, r_i^b$. This quantity can be read at the intersection of the line of slope $-\lambda$ with the ordinate axis.

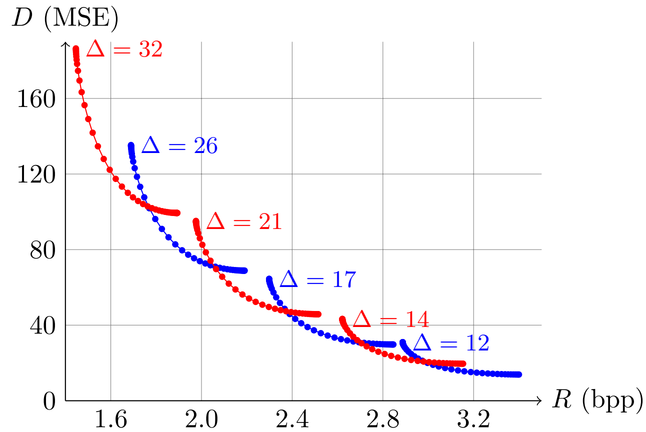

Fig. 3. Rate-distortion curves computed on a 12-bit depth image. Each curve is obtained with a fixed quantization step Δ and with Lagrangian multipliers λ ranging from 1 to 5000. Each dot corresponds to a value of λ.

4.2. Optimal rate distortion curves with fixed quantization steps

Algorithm 1 is used to compute an optimal rate-distortion point given a quantization step Δ and a Lagrangian multiplier λ. The rate-distortion curve optimal for a fixed quantization step Δ is computed with λ ranging from 0 to $+\infty$. In order to obtain the optimal rate-distortion curve for any quantization step Δ, the same process is repeated for many other values Δ. Some of the resulting rate-distortion curves are plotted on figure 3. The lower hull of these curves is the optimal rate-distortion curve for any quantization step.

4.3. Empirical optimal Lagrangian multiplier

In order to verify the accuracy of the theoretical expressions (5) and (8), the best couples (Δ, λ) have to be found among all the previously computed rate-distortion points. Given a Lagrangian multiplier λ, the optimal quantization step Δ is the one such that the rate-distortion cost computed using this couple (Δ, λ) is smaller than the one computed using any other quantization step Δ':

$$\forall \Delta' \quad D(\Delta, \lambda) + \lambda\, R(\Delta, \lambda) \leq D(\Delta', \lambda) + \lambda\, R(\Delta', \lambda)$$

The search of the optimal couples (Δ, λ) also amounts to the search of the lower hull of the rate-distortion curves. The best Lagrangian multiplier is then the opposite of the slope of this hull:

$$\lambda = -\frac{\partial D}{\partial R}$$

Studies have been conducted on six large Earth observation images. Three of them are simulated images of PLEIADES satellite at spatial resolution of 70 cm. PLEIADES first satellite is to be launched in 2010. The targeted bit rate for on-board compression is 2.5 bpp. The other three images have been acquired by PELICAN airborne sensor and have a resolution of 20 cm. As in [1] the 9/7 CDF (Cohen-Daubechies-Feauveau) wavelet transform is used with three levels of decomposition. As the images size is 1024×1024, there are 64512 blocks 4×4 post-processed by image (the low resolution subband is not post-transformed).

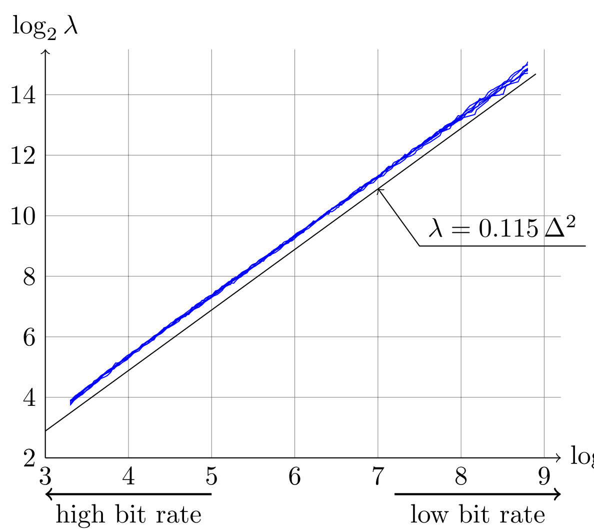

Fig. 4. Optimal Lagrangian multipliers λ computed on six Earth observation images compared to the theoretical curve $\lambda = 0.115\, \Delta^2$.

On figure 4, the optimal Lagrangian multipliers λ obtained on these images are plotted as functions of the quantizer steps Δ. It can be observed that $\lambda(\Delta)$ is a quadratic function. Indeed, the increasing rate of $\log_2 \lambda$ is $2 \times \log_2 \Delta$. Nevertheless, the theoretical values do not fit the experimental curves. They are better approximated by

$$\lambda \approx 0.15\, \Delta^2 \tag{9}$$

as emphasized on figure 5 on which the ratios λ/Δ^2 have been plotted as functions of the bit rate R. It can be seen that this ratio is almost constant for R between 0.2 bpp and 3.5 bpp. The empirical expression (9) is used for compression performance comparisons in section 5.

5. RESULTS ON SATELLITE IMAGES

On figure 6, the optimal compression results obtained by a nearly exhaustive search on the Lagrangian multiplier λ are compared to the

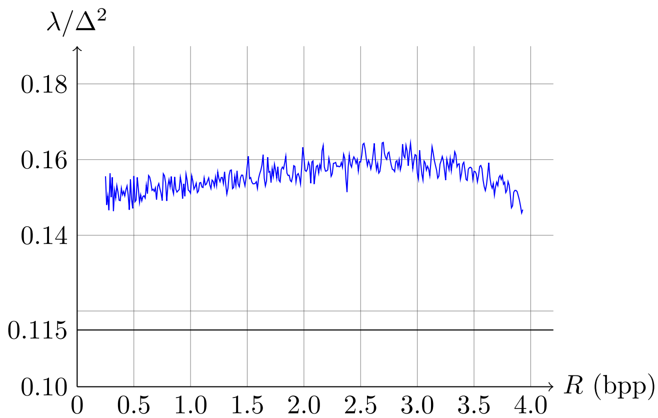

Fig. 5. Mean ratio between the optimal Lagrangian multipliers λ and the square of the quantizer steps Δ^2 on six Earth observation images. This ratio is compared to the theoretical value 0.115 on a wide range of compression bit rates.

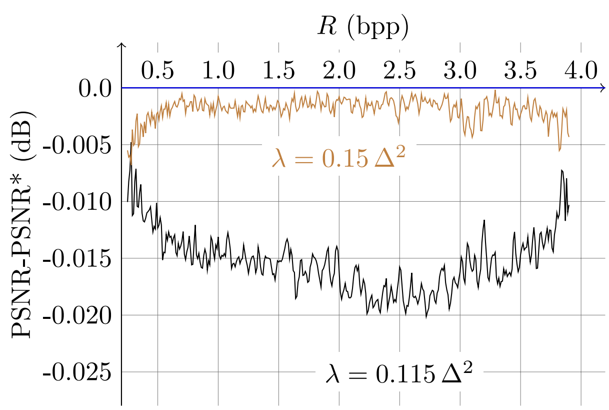

Fig. 6. Losses in PSNR using different formulas to compute the Lagrangian multiplier compared to the best achievable PSNR. These are mean results on six Earth observation images.

results obtained using the theoretical formula $\lambda = 0.115\,\Delta^2$ and the empirical formula $\lambda = 0.15\,\Delta^2$. The formula obtained experimentally always gives better results than the theoretical formula and the optimal results are slightly better than the results obtained using either formulas. Nevertheless, the losses with the formulas are always less than 0,02 dB in PSNR.

Indeed, even if the Lagrangian multipliers are not optimal, the rate-distortion points computed are still optimal for the Lagrangian multipliers used. This is the case for the curves shown on figure 3. Thus, the impact of a small error on the Lagrangian multiplier remains small since the selection of the best basis is still performed through the rate-distortion optimization process of the algorithm 1.

Although the compression results obtained using the theoretical formula of the Lagrangian multiplier are more than satisfactory compared to the optimal compression results, better compression results are obtained with the new empirical formula. This validates the study on the search of the best Lagrangian multiplier for the rate-distortion cost used in the post-transform selection process. Moreover, compression results obtained using another dictionary with different bases are also improved by using the new empirical formula of the Lagrangian multiplier. The optimal Lagrangian multiplier does not depend on the dictionary of bases used.

6. CONCLUSION & PERSPECTIVES

This study has shown that the optimal Lagrangian multipliers for the selection of the best basis for the post-transform can be computed by a nearly exhaustive process. For complexity reasons, it is still highly preferable to use a formula to compute the Lagrangian multiplier even if this formula is sub-optimal. The theoretic formula of the Lagrangian multiplier as a function of the quantization step under the low bit rate hypothesis is surprisingly similar to the theoretic formula obtained under the high resolution assumption. By computation of the optimal compression performance using a nearly exhaustive process, it has been shown that the compression performance obtained using these formulas are close to be optimal. However, another formula has been derived from the experimental results obtained on Earth observation images. This last slightly improves the compression results compared to the use of the theoretical formulas. This improvement may be due to excessive simplifications in in the theoretical formulas.

Since the compression scheme used here directly applies an adaptive arithmetic coder on the post-transformed coefficients, modifications are needed to adapt it to an embedded coder which is highly desirable on-board satellites. Future work will focus on the selection of the best post-transform basis in the context of a bit plane encoder.

7. ACKNOWLEDGMENTS

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8. REFERENCES

[1] CCSDS, *Image Data Compression Recommended Standard CCSDS 122.0-B-1 Blue Book*, Nov. 2005.

[2] D.S. Taubman and M.W. Marcellin, *JPEG2000: Image compression fundamentals, standards and practice*, Kluwer Academic Publishers, 2001.

[3] G. Peyré and S. Mallat, "Discrete bandelets with geometric orthogonal filters," in *IEEE Int. Conf. on Image Proc.*, Sept. 2005, vol. 1, pp. I– 65–8.

[4] X. Delaunay, M. Chabert, V. Charvillat, G. Morin, and R. Ruiloba, "Satellite image compression by directional decorrelation of wavelet coefficients," in *IEEE Int. Conf. on Acoust., Speech and Sig. Proc.*, Apr. 2008, pp. 1193–1196.

[5] Y. Shoham and A. Gersho, "Efficient bit allocation for an arbitrary set of quantizers," *IEEE Trans. on Acoust., Speech, and Sig. Proc.*, vol. 36, no. 9, pp. 1445–1453, Sept. 1988.

[6] K. Ramchandran and M. Vetterli, "Best wavelet packet bases in a rate-distortion sense," *IEEE Trans. on Image Proc.*, vol. 2, no. 2, pp. 160–175, Apr. 1993.

[7] E. Le Pennec and S. Mallat, "Sparse geometric image representations with bandelets," *IEEE Trans. on Image Proc.*, vol. 14, no. 4, pp. 423–438, Apr. 2005.

[8] S-Z. Azimifar, *Image Models for Wavelet Domain Statistics*, Ph.D. thesis, University of Waterloo, Ontario, Canada, 2005.

[9] F. Falzon and S. Mallat, "Analysis of low bit rate image transform coding," *IEEE Trans. on Sig. Proc.*, vol. 46, no. 4, pp. 1027–1042, Apr. 1998.